QISKIT Global Summer School 2020 Notes

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Contents

1 Lecture 1: Qubits and Quantum States, Quantum Circuits, Measurements - Part 1

1.1 From bits to qubits: Dirac Notation, Measurements, Bloch Sphere

- Classical states for computation are either ”0” or ”1”
- In QM, a state can be in superposition, i.e., simultaneously in ”0” and ”1”
  - Superposition’s allow to perform calculations on many gates at the same time.
  - Quantum algorithms with exponential speed-up.
  - **BUT:** once we measure the superpositions state, it will collapse to one of its states.
  - **(we can only get one ”answer” and not all answer to all states in superposition)**
  - It is not that easy to design quantum algorithms, but we use interference effects.
  - **(”wrong answers” cancel each other out, while the ”right answer” remains)**

- **Dirac Notation:**
  - Used to describe quantum states: Let
$$a, b \in \mathbb{C}^2$$
    - ∗ **ket** : $|a\rangle = \binom{a_1}{a_2}$
    - ∗ **bra** : $\langle b| = |b\rangle^t = \binom{b_1}{b_2}^t = \begin{pmatrix} b_1^* & b_2^* \end{pmatrix}$
    - ∗ **bra − ket** : $\langle b|a\rangle = a_1 \times b_1^* + a_2 \times b_2^* = \langle a|b\rangle^*$

- $*$ **ket − bra** $: |a\rangle\langle b| = \begin{pmatrix} a_1 b_1^* & a_1 b_2^* \\ a_2 b_1^* & a_2 b_2^* \end{pmatrix}$

- We define the states:

$$|0\rangle := \begin{pmatrix} 0 & 1 \end{pmatrix}$$

  which are orthogonal:

$$\langle 0|1\rangle = 1 \cdot 0 + 0 \cdot 1$$

- All quantum states are normalized, i.e.,

$$\langle \psi|\psi\rangle = 1$$

  e.g.

$$|\psi\rangle = \frac{1}{\sqrt{2}} \cdot (|0\rangle + |1\rangle) = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

- **Measurements:**
  - We choose orthogonal bases to describe and measure quantum states (projective measurement)
  - During a measurement onto the basis $\{\ |0\rangle, |1\rangle\}$, the state will collapse into either state $|0\rangle$ or $|1\rangle$ as those are the eigenstates of σ_z, we call this a Z-measurement
  - There are infinitely many different bases, but other common ones are: $\{\ |+\rangle := \frac{1}{\sqrt{2}}, |-\rangle := \frac{1}{\sqrt{2}}\ \}$ and $\{\ |+i\rangle := \frac{1}{\sqrt{2}}, |-i\rangle := \frac{1}{\sqrt{2}}\}$ corresponding to the eigenstates of σ_x and σ_y.

  - **Born Rule:** The probability that a state $|\psi\rangle$ collapses during a projective measurement onto the basis $\{|x\rangle, |x\rangle\}$ to the state $|x\rangle$ is given by:

$$P(x) = |\langle x|\psi\rangle|^2$$

$$\sum_i P(x_i) = 1$$

    - $*$ **Examples:**
      - $\cdot$ $|\psi\rangle = \frac{1}{\sqrt{3}}(|0\rangle + \sqrt{2}|1\rangle)$ is measured in the basis $\{|0\rangle\ ,\ |1\rangle\}$:

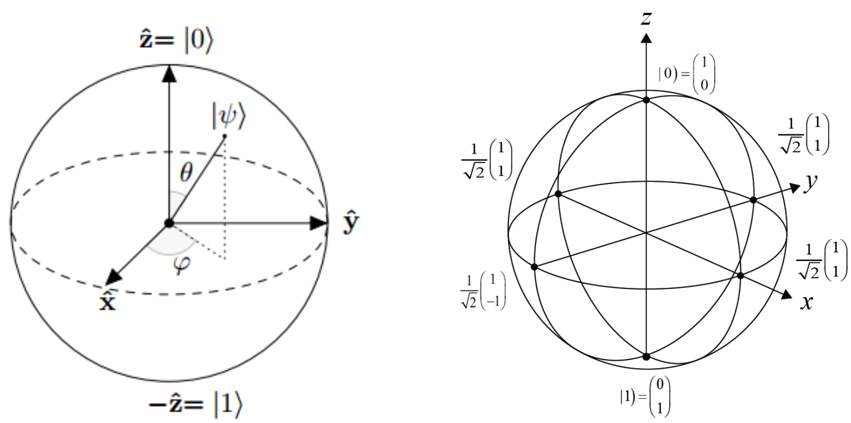
$$P(0) = \left|\langle 0| * \frac{1}{\sqrt{3}}\,(|0\rangle + \sqrt{2} \cdot |1\rangle)\right|^2 = \left|\frac{1}{\sqrt{3}} \cdot \langle 0|0\rangle + \frac{\sqrt{2}}{\sqrt{3}} \cdot \langle 0|1\rangle\right|^2 = \frac{1}{3}\ ;\ P(1) = \frac{2}{3}$$

      - $\cdot$ $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ is measured in the basis $\{\ |+\rangle\ ,\ |-\rangle\ \}$:

$$P(+) = |\langle +|\psi\rangle|^2 = \left|\frac{1}{\sqrt{2}} \cdot (\langle 0| + \langle 1|) + \frac{1}{\sqrt{2}} \cdot (|0\rangle - |1\rangle)\right|^2$$

$$= \frac{1}{4}\left|\langle 0|0\rangle - \langle 0|1\rangle + \langle 1|0\rangle - \langle 1|1\rangle\right|^2 = 0$$

        expected as $\langle +|\psi\rangle = \langle +|-\rangle = 0$



(a) (b)

Figure 1: Bloch Sphere

- **Bloch Sphere:**
  - We can write any normalized(pure) state as $|\psi\rangle = \cos\frac{\theta}{2}\cdot|0\rangle + e^{\iota\phi}\sin\frac{\theta}{2}\cdot|1\rangle$, where $\phi \in [0, 2\pi]$ describes the relative phase and $\theta \in [0, \pi]$ determines the probability to measure $|0\rangle$ / $|1\rangle$:

$$P(|0\rangle) = \cos^2\frac{\theta}{2}\ ,\ \ P(|1\rangle) = \sin^2\frac{\theta}{2}$$

    - All normalized pure states can be illustrated on the surface of a sphere with radius $\vec{r}$, which we call the **Bloch Sphere**
    - The coordinates of such a state are given by the **Bloch Vector**:

$$\vec{r} = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

  - **Examples:**

$$|0\rangle : \theta = 0,\ \phi\ \ is\ arbitary \rightarrow \vec{r} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad |1\rangle : \theta = \pi,\ \phi\ \ is\ arbitary \rightarrow \vec{r} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$|+\rangle : \theta = \frac{\pi}{2},\ \phi = 0\ \rightarrow \vec{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad |-\rangle : \theta = \frac{\pi}{2},\ \phi = \pi\ \rightarrow \vec{r} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

$$|+i\rangle : \theta = \frac{\pi}{2},\ \phi = \frac{\pi}{2}\ \rightarrow \vec{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad |-i\rangle : \theta = \frac{\pi}{2},\ \phi = \frac{3\pi}{2}\ \rightarrow \vec{r} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

  - **<u>Be Careful:</u>** On the Bloch sphere, angles are twice as big as in Hilbert Space, e.g. $|0\rangle$ $|1\rangle$ are orthogonal, but on the Bloch sphere their angle is 180deg $.Forageneralstate—\psi\rangle = \cos\frac{\theta}{2}\,|0\rangle + ...$
    Here θ is the angle on the Bloch Sphere, while $\frac{\theta}{2}$ is the actual angle in the Hilbert Space!

  - Z-measurement corresponds to a projection onto the z-axis and analogously for X and Y!

1.2 Quantum Circuits : basic single-qubit and two-qubit gate, multi-partite quantum states

1.3 Entanglement: Bell states, Teleportation, Q-shpere

Examples include:

- the **double slit experiment** (done with light by Thomas Young in 1801, and with electrons by Tonomura in 1986)
- the photoelectric effect (analyzed by Einstein in 1905 — in fact his Nobel-winning work)
- the "quantum Venn diagram" puzzle, involving the overlaps of three polarizing filters
- the stability of the hydrogen atom (i.e., the fact that the electron doesn't lose energy and spiral inward toward the proton).

Remark 1. *How now, brown cow?*

Definition 1. *The* Feynman kernel *is given by*

$$K(x_b, t_b; x_a, t_a) = \int_{x(t_a)=x_a}^{x(t_b)=x_b} e^{(i/\hbar)S[x(t)]}\ \mathscr{D}x(t).$$

1.4 Mon, Sept 9: Review of Newtonian Mechanics

- A *Newtonian trajectory* $\mathbf{x}(t)$ ($t \in \mathbb{R}$) is given by solutions of the second order ODE

  $$m\,\ddot{\mathbf{x}}(t) = \mathbf{F}(\mathbf{x}(t)),$$

  where $m > 0$ is a basic parameter associated with a given Newtonian particle, called its *mass*.
- The force field $\mathbf{F}(\mathbf{x})$ — which we take to be static (i.e., not intrinsically dependent on time) for simplicity — is said to be *conservative* if there is a *potential function* $V(\mathbf{x})$ such that

  $$\mathbf{F}(\mathbf{x}) = -\nabla V(\mathbf{x}).$$

  Here, '∇' denotes the *gradient operator*,

  $$\nabla V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right).$$
- For a conservative force field, we can find a *conserved quantity* along the Newtonian trajectories, namely the *total mechanical energy*.

  $$E = H(\mathbf{x}, \mathbf{p}) := \frac{1}{2m}\,\mathbf{p}^2 + V(\mathbf{x}).$$

  Here, $\mathbf{p}^2 := \mathbf{p} \cdot \mathbf{p} = \|\mathbf{p}\|^2$, and $\mathbf{p} := m\mathbf{v} := m\dot{\mathbf{x}}$ is the *momentum*.

1.5 Tue, Sept 10: Alternative Formulations of Newtonian Mechanics

- The **Hamiltonian formulation**:

  $$\dot{\mathbf{x}} = \frac{\partial H}{\partial \mathbf{p}}, \qquad \dot{\mathbf{p}} = -\frac{\partial H}{\partial \mathbf{x}}.$$
- The **Lagrangian formulation**:

  $$\delta S[\mathbf{x}(t)] = 0,$$

  where the *action* on the time interval $[t_a, t_b]$ is given by

  $$S[\mathbf{x}(t)] := \int_{t_a}^{t_b} \left[\frac{m}{2}\,\dot{\mathbf{x}}(t)^2 - V(\mathbf{x}(t))\right] dt.$$
- Etc.

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